

A differential equation approach for examining the subtraction schemes

Ji-Feng Yang*

School of Management, Fudan University, Shanghai 200433, P R China

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We propose a natural differential equation with respect to mass(es) to analyze the scheme dependence problem. It is shown that the vertex functions subtracted at an arbitrary Euclidean momentum (MOM) do not satisfy such differential equations, as extra unphysical mass dependence is introduced which is shown to lead to the violation of the canonical form of the Slavnov-Taylor identities, a notorious fact with MOM schemes. By the way, the traditional advantage of MOM schemes in decoupling issue is shown to be lost in the context of Callan-Symanzik equations.

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Conventionally, it is widely believed that different renormalization schemes (that differ arbitrarily with each other) are perfectly equivalent provided the perturbation is completed. Once truncated at finite order, they yield the annoying scheme dependence problem [1], i.e., different schemes yield different predictions, and we have no better reasons to prefer one scheme to the others. But recently, it has been realized that the traditional on-shell definition of unstable particles masses and width (say, e.g., masses and widths of W^\pm, Z^0 and Higgs particles in the electroweak model) is problematic and should be replaced by the more physical and appropriate pole mass schemes [2], as the on-shell schemes would incur severe gauge dependence and IR singularities or ambiguities. That means, though we do not know the unique way of performing the physical definition, the freedom in choosing renormalization conditions or schemes deserves closer examination.

In this brief report, we suggest to examine the problem by formulating the Feynman amplitudes as a series of differential equations parametrized in terms of momenta (and masses for certain theories)*, this is feasible since the loop amplitudes must be functions of external momenta and masses after all. Then, as we shall see shortly, not all subtraction prescriptions or schemes satisfy such differential equations. The differential equation analysis follows from the well known fact that the differentiation with respect to physical parameters like momenta and masses lowers the divergence degree of a superficially divergent Feynman diagram [4]. The solutions to these natural differential equations must be well defined or finite functions of momenta (and masses if any) and uniquely defined up to a certain set of constants to be defined through 'boundary' conditions, which just corre-

spond to the definition of a renormalization/subtraction scheme, the parameters of the solutions must have been finite ones. It is worthwhile to point out that such analysis has already applied in the usual renormalization programs through the critical use of Ward identities, a set of partial differentiation equations in terms of momenta. Our differential equation analysis is in fact a mathematical generalization of the differential form Ward identities. In this short report, we temporarily focus on the differential equations in terms of masses for QFTs with massive fields.

One might object such analysis with respect to masses as the differentiation operation would shift or perturb the original theory. Such an objection is simply unjustified. First, no matter what kind of changes the differentiation brings into the theory, it does bring us information between the two theories that slightly differ in masses. Then as long as the difference is essentially analytical, we can integrate the difference back to find mass dependence of the original theory (up to ambiguities to be fixed by boundary conditions). Second, a more direct and mathematical way to see this is, if we believe that the theories depend on the mass parameters in an almost analytical way, or that the Feynman diagrams are regular functions of masses, then no matter what the differentiation with respect to masses physically brings about, we can perform differential analysis. Until now, we have not learned of any example of QFT whose mass dependence is so singular that it defies differentiation equation analysis. So, as far as the mass dependence of Feynman amplitudes is concerned, it does not matter whether or not a method shifted the original theory or violated any symmetry of the original theory as long as the perturbation or violation give rise to no completely singular mass dependence. We do not know any special roles played by the gauge symmetry and SUSY in preserving the analytical mass dependence.

In fact such operation preserves most novel symmetries, like gauge invariance, Lorentz invariance, and SUSY since canonical masses are gauge invariant, Lorentz invariant and SUSY invariant if they are masses of SUSY multiplet. For masses from symmetry breaking,

*Viewing the quantum field theories as just effective sectors of the complete and well defined quantum theory of everything (QTOE), we can derive finite loop integrals by formulating them as solutions of appropriate differential equations in terms of energy, momenta and masses, without introducing any UV divergences, see Ref. [3].

the differentiation equation analysis will become complicated due to the complexity of the Higgs sector which will be examined in a separate report. Here we focus on the invariant mass cases to demonstrate that not all subtraction schemes are consistent with the differential equation analysis. Note that we are not claiming that these subtraction schemes can not consistently renormalize QFTs with masses. We leave the implications of our result open to the readers.

We will illustrate the analysis with a simple vertex function at the lowest order in a QFT, say, the 1-loop photon vacuum polarization tensor $\Pi^{\mu\nu}(p, -p, m)$ ($\equiv -ie^2 \int d^n k \text{tr}\{\gamma^\mu \frac{1}{p+k-m} \gamma^\nu \frac{1}{k-m}\}$) in QED for simplicity. But the main points of the analysis apply in the same way to higher orders and to other theories like massive scalar theories and heavy quark theories.

It is easy to see that this amplitude satisfies the following well defined inhomogeneous differential equation in any gauge invariant regularization scheme (*GIR*)

$$\begin{aligned} & \partial_m \Pi^{\mu\nu}(p, -p, m) \\ &= -ie^2 \int (d^4 k)_{\text{GIR}} \text{tr}\{\gamma^\mu \left(\frac{1}{p+k-m}\right)^2 \gamma^\nu \frac{1}{k-m} \\ &+ \gamma^\mu \frac{1}{p+k-m} \gamma^\nu \left(\frac{1}{k-m}\right)^2\} \end{aligned} \quad (1)$$

as the RHS of Eq.(1) is well defined in such schemes. Or it satisfies the following well defined equation in any regularization schemes

$$\begin{aligned} & (\partial_m)^3 \Pi^{\mu\nu}(p, -p, m) = -ie^2 \sum_{l=0}^3 C_l^3 \int (d^4 k)_{\text{GIR}} \\ & \text{tr}\{\gamma^\mu \left(\frac{1}{p+k-m}\right)^{l+1} \gamma^\nu \left(\frac{1}{k-m}\right)^{4-l}\} \end{aligned} \quad (2)$$

where C_l^3 is the combinatorial factor arising from the differentiation operation.

It suffices to demonstrate our points with Eq.(1). Dropping the mass independent transverse factor $(g^{\mu\nu} p^2 - p^\mu p^\nu)$ we have the following equation

$$\partial_m \Pi(p^2, m) = \frac{e^2}{\pi^2} \int_0^1 dx \frac{x(1-x)m}{m^2 - x(1-x)p^2}. \quad (3)$$

The solution to this equation is easy to find, it reads

$$\begin{aligned} \Pi(p^2, m; C) = & \frac{e^2}{2\pi^2} \int_0^1 dx x(1-x) \ln \frac{m^2 - x(1-x)p^2}{C} \\ & + F(p^2) \end{aligned} \quad (4)$$

with C being the natural integration constant independent of to be fixed by regularization and/or subtraction schemes and $F(p^2)$ denoting an arbitrary mass independent function of momentum which can be fixed to be zero by solving the differential equation in terms of momentum. Then we have

$$\begin{aligned} & \Pi^{\mu\nu}(p, -p, m; C) \\ &= \frac{e^2}{2\pi^2} (g^{\mu\nu} p^2 - p^\mu p^\nu) \int_0^1 dx (x - x^2) \ln \frac{m^2 - (x - x^2)p^2}{C} \end{aligned} \quad (5)$$

with mass here being a finite or renormalized one. Note that the constant C here denotes any regularization scheme that is both gauge invariant and mass independent. In other words, those possible mass dependent regularization schemes are already excluded, just like we exclude those regularization schemes that violate gauge invariance and other physical symmetries in realistic models.

In all mass independent schemes, the subtraction is so defined that C is fixed to be a mass independent constant which still satisfies Eq.(3). However, in the MOM schemes, the subtraction is defined in the following way,

$$\begin{aligned} & \Pi(p^2, m; C_{\text{Mom}}) \|_{p^2 = -\mu^2} = 0 \\ & \Rightarrow C_{\text{Mom}} = m^2 + x(1-x)\mu^2 \end{aligned} \quad (6)$$

which ceases to be a solution of Eq.(3) but satisfies the following equation instead

$$\begin{aligned} \partial_m \Pi(p^2, m; C_{\text{Mom}}) = & \frac{e^2}{\pi^2} \int_0^1 dx \frac{x(1-x)m}{m^2 - x(1-x)p^2} \\ & + \delta(\mu^2, m), \end{aligned} \quad (7)$$

with $\delta(\mu^2, m) \equiv -\frac{e^2}{\pi^2} \int_0^1 dx \frac{x(1-x)m}{m^2 + x(1-x)\mu^2}$. It is clear in Eq.(7) that the canonical relation of Eq.(3) is violated by the presence of *an extra term that is dependent on the arbitrary subtraction point μ* when massive particles are present. The extra term comes from the nontrivial dependence of subtraction constant upon particle mass in contrast to the mass independent schemes (MS) [5] where $\partial_m C_{\text{MS}} \equiv 0$, and therefore becomes the source of problems in realistic applications. Regarding the mass dependence, the on-shell scheme belongs to the same class as MOM. It will lead to 'extra unphysical' mass dependence for the vertex functions (including self-energies) for theories with massive bosons, which is just the source of troubles illustrated in [2]. The MOM like schemes are also known to violate the canonical Ward or Slavnov-Taylor identities except in the background gauge [6], which we think is related to the fact pointed out above.

We would like to provide more arguments regarding the mass dependence of renormalization schemes. Since the mass differentiation would insert mass operators that is closely related to the trace of the energy stress tensor which in turn couples to gravity, such operation should not lead to any 'extra unphysical' piece. It is easy to see that in Eq.(3) we can replace ∂_m by $m\partial_m$ without changing the problem. Alternatively, the $m\partial_m$ operation is part of the overall rescaling operation which also includes $p^\mu \partial_{p^\mu}$. The latter operation should unveil the physical dependence of vertex functions on spacetime, and

hence the mass dependence of vertex functions should also be physical due to the overall rescaling entangle the mass dependence with that of momenta. We can then understand why mass dependent schemes often violate the canonical Ward identities, as the differential form of Ward identities contain the operation ∂_{p^μ} , if we multiply this operator from the left by p^μ then we see that it is part of the scaling equations. Since the overall homogeneity of the vertex functions with respect to all massive variables will entangle the momentum dependence with the mass dependence, the extra term in Eq.(7) will lead to violation of the canonical Ward identities. In this connection we note that the pole mass for fermion is related to the effective mass which is gauge invariant and IR finite [7,8] by the equation $m_{eff}(p^2, \bar{m})|_{p^2=M_{pol}^2} = M_{pol}$ [8], i.e., the dependence of pole mass on the MS running mass (Lagrangian mass) is *normal* since m_{eff} can be defined in MS scheme though it is scheme independent [9]. The situation for bosons is similar, see [2].

Thus, contrary to the naive expectation, the differential equation analysis does not allow for all kinds of subtraction schemes. That means we should reexamine our conventional belief with due care. If we accepted this approach, then the conventionally claimed scheme invariance [1] of certain objects within a subset of schemes (i.e., within the mass independent schemes) can now be viewed as a full 'invariance' for all schemes that solve the differential equations in terms of all physical parameters. Otherwise, the significance of the claims like the significance of scheme invariance of the first two coefficients (β_0, β_1) of beta function would be diminished by the mass dependent schemes. In a sense we advanced a plausible rationale for using mass independent schemes in our calculations. We also wish to point out that our investigation in fact derives rationale from the well known fact that not all regularization schemes would satisfy the Ward identities. In our point of view, complicated schemes (like MOM) not only bring about inconvenience, but also might be unnatural choices. In other words, the freedom in choosing the schemes is a restricted freedom if one works in the differential equation approach suggested above.

Indeed there exists no literature up to date that provided convincing constraints on mass dependence of schemes, but it is also hard to convince people that a scheme that defines the one loop vacuum polarization tensor like below would be a physically acceptable choice:

$$\begin{aligned} & \Pi^{\mu\nu}(p, -p, m; \mu) \|_{Toy} \\ &= \frac{e^2}{2\pi^2} (g^{\mu\nu} p^2 - p^\mu p^\nu) \left\{ \int_0^1 dx x(1-x) \ln \frac{m^2 - x(1-x)p^2}{m^2} \right. \\ & \quad \left. + \left(\frac{m^2}{\mu^2 + m^2} \right)^{1000} \right\}, \end{aligned} \quad (8)$$

that is to say, such a subtraction leaving a finite term like $\left(\frac{m^2}{\mu^2 + m^2} \right)^{1000}$ can not be an acceptable choice, contrary to

the belief that the subtraction can be arbitrary as long as divergences are removed, at least the anomalous dimension will be awkward-looking and very different from the standard results. In fact we can consider the problem from the reverse angle, i.e., suppose we found the true physical and hence scheme independent parametrization, these physical parameters (together with a RG and renormalization scheme invariant scale Λ_{phys}) work coherently to give the unique mathematical formulas that are often complicated functions. Then it is impossible to transform these physical parameters into arbitrary things beyond novel symmetries or invariances, i.e., it is impossible to redefine the parameters in arbitrary schemes, since we must confront our calculations with experiments and experiments will only justify the schemes in which the dependence of the measured quantities (defined as functionals of Feynman amplitudes) upon the renormalized parameters are not quite different from the dependence upon physical parameters, as the renormalized parameters functionally take the same role as the physical ones, an obvious fact in the formulation of QFT. Our investigation above seems to add doubts to the conventional attitude to this issue. Since it is just a first attempt, we will refrain from claim anything, we only wish to draw attention to the above aspects.

The differential equations can be generalized to any order and any QFT with massive fields:

$$\begin{aligned} & (\partial_{m_i^s})^\omega \Gamma(p_1, \dots, (m_i^s); [C]) \\ &= \Gamma_{\frac{\phi_1^2}{s}, \dots} (p_1, [0]_\omega, \dots, (m_i^s); [C']), \quad s = 1, 2, \quad \omega \geq 1, \end{aligned} \quad (9)$$

where (m_i^s) refers to the various masses in the theory with $s = 1$ for fermion and $s = 2$ for boson, $[C]$ and $[C']$ refer to the constants that will appear in the solutions and ω being any times that will annihilate some of the constants, which is the secret of the analysis in mass differentiation. Note that we have made use of the fact that the differentiation with respect to mass(es) inserts the mass vertex operators—the operator (ϕ_j^2/s) (for fermion it should be understood as $\bar{\psi}\psi$).

Conventionally the MOM schemes are held to be advantageous over the mass independent schemes in exhibiting good decoupling behavior required by physics. In this connection, we wish to prove that in the context of Callan-Symanzik equation [11] the decoupling of heavy fields [10] is achieved in the same way in both mass independent schemes and mass dependent schemes. Especially, we do not need the so-called "effective field theories" framework [12] to help the mass independent schemes, which is needed in the context of renormalization group equation (RGE) since it does not account for the effects of *all* the dimensional parameters. Such a proof is not seen yet in literature. Again we will illustrate it with a simple model, QED with a massive fermion in addition to n_l massless fermions. In a mass independent scheme the Callan-Symanzik equation reads,

$$\begin{aligned} & \{\lambda\partial_\lambda - \beta\alpha\partial_\alpha + \gamma_\Gamma - D_\Gamma\}\Gamma((\lambda p), m, \alpha, \mu) \\ &= -i\Gamma^\Theta((\lambda p), m, \alpha, \mu) \end{aligned} \quad (10)$$

where $\Theta \equiv [1 + \gamma_m]m\bar{\psi}\psi$, β , γ_Γ and γ_m are mass independent functions of the renormalized coupling α and all quantities are renormalized ones. At lowest order, $\beta = \frac{2\alpha}{3\pi}[n_l + 1]$.

When the mass goes to infinity, it is natural to expect that

$$\begin{aligned} & -i\Gamma^\Theta((p), m, \alpha, \mu)|_{m \rightarrow \infty} \\ &= (\Delta\beta\alpha\partial_\alpha - \Delta\gamma_\Gamma)\Gamma((p), \alpha, \mu), \end{aligned} \quad (11)$$

$$\begin{aligned} & \rightarrow \{\lambda\partial_\lambda - \beta_{ls}\alpha\partial_\alpha + \gamma_{\Gamma;ls} - D_\Gamma\}\Gamma((\lambda p), \alpha, \mu)|_{ls.} = 0. \\ & \end{aligned} \quad (12)$$

Here $\beta_{ls} \equiv \beta + \Delta\beta$, $\gamma_{\Gamma;ls} \equiv \gamma_\Gamma + \Delta\gamma_\Gamma$ with the delta contributions coming from the mass insertion part in the infinite mass limit that will cancel the heavy fields' pieces in β and γ . The generalization to other theories with boson masses is an easy exercise. From Eq.(12) we see that the decoupling of heavy particles is realized in a natural way in the contexts of Callan-Symanzik equations.

To verify the above deduction it is enough to demonstrate Eq.(11) at the lowest order which is closely related to the observation that heavy particle limit provides a convenient algorithm for calculating trace anomalies [13]

$$\begin{aligned} & -i\langle m\bar{\psi}\psi J^\mu J^\nu \rangle|_{m \rightarrow \infty} = \frac{2\alpha}{3\pi}(p^2 g^{\mu\nu} - p^\mu p^\nu) \\ & \Rightarrow m(1 + \gamma_m)\bar{\psi}\psi|_{m \rightarrow \infty} = \frac{1}{4}\Delta\beta F^{\mu\nu}F_{\mu\nu}, \end{aligned} \quad (13)$$

with $J^\mu \equiv -ie\bar{\psi}\gamma^\mu\psi$ and $\Delta\beta \equiv -\frac{2\alpha}{3\pi} = \Delta\gamma_A$. When translated into Callan-Symanzik equations Eq.(13) is just Eq.(11). The cancellation of the heavy particle contributions is obvious since $\beta + \Delta\beta = \frac{2\alpha}{3\pi}[n_l + 1] - \frac{2\alpha}{3\pi} = \frac{2\alpha}{3\pi}n_l$.

While in the MOM schemes the Callan-Symanzik equation reads,

$$\begin{aligned} & \{\lambda\partial_\lambda - \beta_{Mom}\alpha\partial_\alpha + \gamma_{\Gamma;Mom} - D_\Gamma\}\Gamma_{Mom}((\lambda p), m, \alpha, \mu) \\ &= -i\Gamma_{Mom}^\Theta((\lambda p), m, \alpha, \mu) \end{aligned} \quad (14)$$

with the beta function,etc. being defined as $\beta_{Mom} \equiv (\mu\partial_\mu + m(1 + \gamma_{m;Mom})\partial_m)\alpha, \dots$, in contrast to the RGE definition: $\beta_{Mom}^{RG} \equiv \mu\partial_\mu\alpha, \dots$, due to the mass dependence of the renormalization constants[†]. The resulting β_{Mom} also exhibits nondecoupling feature as in the mass

independent schemes. For example, at the lowest order, from the definition given above, a heavy particle's contribution to β at lowest order is

$$\begin{aligned} \beta_{Mom} &= (\mu\partial_\mu + m\partial_m)\frac{e^2}{2\pi^2} \int_0^1 dx x(1-x) \ln C_{Mom} \\ &= \frac{2\alpha}{3\pi}. \end{aligned} \quad (15)$$

Then it is easy to find what is similar to Eq.(11)

$$\begin{aligned} & -i\Gamma_{Mom}^\Theta((p), m, \alpha, \mu)|_{m \rightarrow \infty} \\ &= (\Delta\beta_{Mom}\alpha\partial_\alpha - \Delta\gamma_{\Gamma;Mom})\Gamma_{Mom}((p), \alpha, \mu)|_{ls}, \end{aligned} \quad (16)$$

with $\Delta\beta_{Mom} = -\frac{2\alpha}{3\pi}$ as Eq.(13) is also true in the MOM schemes at the lowest order. It is known that the first loop order beta of RGE in mass independent schemes differs from that in the MOM schemes [9]. While in the context of Callan-Symanzik equation the first loop order beta function is the same in all schemes at one loop order (C.f. Eq.(15)), which in turn implies that the same decoupling mechanism works in MOM schemes in the context of Callan-Symanzik equation. Thus the advantage of MOM schemes over the mass independent schemes in the decoupling issue is lost in the context of Callan-Symanzik equation and the decoupling is realized in the same way specified by Eq.(11).

We should note that, many MOM scheme calculations of couplings [14] are in fact calculations of the effective couplings with the momentum transfer defined at an Euclidean point. The CWZ [15] scheme for calculations in the presence of heavy quarks is in fact a \overline{MS} scheme with the running scale fixed to be the particle mass, since in this scheme the mass also serve as a running scale for massive loops and the β function, etc. are all mass independent. We should also note that our analysis only shows that the mass dependent schemes like MOM and on-shell schemes *might* cause problems for massive and some related sectors (like self-energy vertices,etc.) in a QFT. For massless sectors or massless QFTs, the problem does not materialize.

If one adopts our proposal for renormalization mentioned above [3], then the decoupling of heavy particles

of the β , γ etc. are the same as in RGE. In this connection, we note that unfortunately many people have confused CSE with RGE which are different things in principle, CSE describes full scaling behavior while RGE only states finite renormalization effects. In addition, in the standard form of CSE, the trace of stress tensor is singled out, which describes the physical coupling of matters to gravity and therefore should be independent of renormalization schemes. Then we can anticipate that the heavy particle limit of this term should leave the same effects to light sectors in any renormalization scheme, i.e., the same cancelling contributions to the full β , γ etc., just as what we will demonstrate here.

[†]In the standard form of Callan-Symanzik equation, the mass operators inserted vertex functions appear separately and the definitions of β , γ in the MOM like schemes must include $m(1 + \gamma_{m;Mom})\partial_m$ to account for the 'full running' of the relevant renormalization constants. But in the alternative form of Callan-Symanzik equation (CSE), i.e., $\{\lambda\partial_\lambda - \beta\alpha\partial_\alpha + m(1 + \gamma_m)\partial_m + \gamma_\Gamma - D_\Gamma\}\Gamma((\lambda p), m, \alpha, \mu) = 0$, the definitions

can be achieved in an automatic way as our proposal stands on the philosophical ground of the effective field theories [12], please see Ref. [16].

In summary, we suggested a differential equation analysis of the radiative corrections in terms of momenta and masses and demonstrated that only certain subtraction schemes satisfy such equations. Some conventional schemes like MOM failed to solve these equations. By the way, we gave a demonstration that in the context of the Callan-Symanzik equations all the schemes facilitate the decoupling of heavy particles in the same way, weakening the argument that the MOM like schemes is superior to mass independent schemes in decoupling issue.

- [13] Ji-Feng Yang, PhD Thesis, Fudan University, unpublished, 1994; G.-j. Ni and Ji-Feng Yang, *Phys. Lett. B* **393**, 79 (1997).
- [14] F. Jegerlehner and O.V. Tarasov, *Nucl. Phys. B* **549**, 481 (1999); S. J. Brodsky, M. Melles and J. Rathsman, *Phys. Rev. D* **60**, 096006 (1999) and references therein.
- [15] J. C. Collins, F. Wilczek and A. Zee, *Phys. Rev. D* **18**, 242 (1978).
- [16] Ji-Feng Yang, Report. No. hep-th/9908111; Ji-Feng Yang, in preparation.

- [1] P. M. Stevenson, *Phys. Rev. D* **23**, 2916 (1981); G. Grunberg, *Phys. Rev. D* **29**, 2315 (1984); S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, *Phys. Rev. D* **28**, 228 (1983); D. T. Barclay, C. J. Maxwell and M. T. Reader, *Phys. Rev. D* **49**, 3480 (1994).
- [2] See, e.g., S. Willenbrock and G. Valencia, *Phys. Lett. B* **259**, 373 (1991); R. G. Stuart, *Phys. Lett. B* **262**, 113 (1991), **272**, 353 (1991), *Phys. Rev. Lett.* **70**, 3193 (1993); A. Sirlin, *Phys. Rev. Lett.* **67**, 2127 (1991), *Phys. Lett. B* **267**, 240 (1991); T. Bhattacharya and S. Willenbrock, *Phys. Rev. D* **47**, 4022 (1993); H. Veltman, *Z. Phys. C* **62**, 35 (1994); M. Passera and A. Sirlin, *Phys. rev. Lett.* **77**, 4146 (1996), *Phys. Rev. D* **58**, 113010 (1998); B. A. Kniehl and A. Sirlin, *Phys. Rev. Lett.* **81**, 1373 (1998), *Phys. Lett. B* **440**, 136 (1998).
- [3] Ji-Feng Yang, Report No. hep-th/9708104; invited talk, 'Can QFT be UV finite as well as effective?', pp202-206 in *Proceedings of the XIth International Conference "Problems of Quantum Field Theory"* (Dubna, Russia, July 13-17, 1998), Eds.: B. M. Barbashov, G. V. Efimov and A. V. Efremov. Publishing Department of JINR, Dubna, 1999(hep-th/9901138); hep-th/9904055.
- [4] E. Witten, *Nucl. Phys. B* **104**, 445 (1976); W. E. Caswell and A. D. Kennedy, *Phys. Rev. D* **25**, 392 (1980).
- [5] G. 't Hooft, *Nucl. Phys. B* **62**, 444 (1973); W. A. Bardeen, *et al*, *Phys. Rev. D* **18**, 3998 (1978); S. Weinberg, *Phys. Rev. D* **8**, 3497 (1973).
- [6] A. Rebhan, *Z. Phys. C* **30**, 309 (1986) and references therein.
- [7] A. S. Kronfeld, *Phys. Rev. D* **58**, 051501 (1998).
- [8] R. Tarrach, *Nucl. Phys. B* **183**, 384 (1981); N. Gray, *et al*, *Z. Phys. C* **48**, 673 (1990).
- [9] R. Coquereaux, *Ann. Phys.* **125**, 401 (1980).
- [10] T. Appelquist and J. Carazzone, *Phys. Rev. D* **11**, 2856 (1975).
- [11] C. G. Callan, Jr., *Phys. Rev. D* **2**, 1541 (1970); K. Symanzik, *Comm. Math. Phys.* **18**, 227 (1970).
- [12] S. Weinberg, *Phys. Lett. B* **91**, 51 (1980); L. J. Hall, *Nucl. Phys. B* **178**, 75 (1981); B. Ovrut and H. Schnitzer, *Nucl. Phys. B* **179**, 381 (1981), **189**, 509(1981).